RWTH Aachen University **WS 2010/11**
Lehrgebiet Theoretische Informatik Tutorial 6
Kneis—Langer—Sikdar 23.11.2010

Tutorial Exact Algorithms

Exercise T12 TSP IN BOUNDED DEGREE GRAPHS. In class, we saw that n-vertex graphs with maximum degree Δ have at most

$$[(2^{\Delta+1} - 1)^{1/(\Delta+1)}]^n$$

connected vertex sets. This is better than the trivial 2^n bound for general graphs: for $\Delta = 3$, for instance, this bound works out to 1.97^n. However the algorithm requires a data structure that stores connected set families of size i, where $1 \leq i \leq n$, that supports the following operations:

- insert a new connected set in polynomial time (polynomial in n, where n is the number of vertices);
- output all connected sets of a particular size in time proportional to their total size;
- given a connected set S, we should be able to read off $\mathrm{OPT}[S; c_i]$ for all $c_i \in S$ in time $O(|S|)$.

In addition, the size of the data structure should be proportional to the total size of all the connected sets. Design an appropriate data structure for this algorithm.

Exercise T13 MINIMUM EXACT SET COVER. This problem is defined as follows: given a finite set $\mathcal{U} = \{1, \ldots, n\}$ and a collection $\mathcal{F}$ of $m > 0$ subsets of $\mathcal{U}$, you have to find a subset $\mathcal{F}' \subseteq \mathcal{F}$ of minimum size that partitions $\mathcal{U}$. That is, the sets in $\mathcal{F}'$ must be pairwise disjoint and their union must be the whole set $\mathcal{U}$. Design a DP algorithm for this problem with running time $O(2^n \cdot nm)$.

Exercise T14 LONGEST COMMON SUBSEQUENCE. Let Σ be a finite alphabet and let $\mathbf{x} = x_1 x_2 \ldots x_n$ and $\mathbf{y} = y_1 y_2 \ldots y_m$ be strings from Σ^*. A subsequence of $\mathbf{x}$ is a string of characters $x_{i_1} x_{i_2} \ldots x_{i_p}$ from the sequence $\mathbf{x}$ such that $i_1 < i_2 < \cdots < i_p$. For example, if $\mathbf{x}$ = `geranium` then `germ` is a subsequence of $\mathbf{x}$. A sequence that is a subsequence of both $\mathbf{x}$ and $\mathbf{y}$ is a *common subsequence*. A common subsequence of maximum length is a *longest common subsequence* of $\mathbf{x}$ and $\mathbf{y}$. Design a DP algorithm to find out the longest common subsequence of $\mathbf{x}$ and $\mathbf{y}$ in time $O(mn)$.

Homework Assignment H13 (10 Points) Consider the EXACT SAT problem: given a Boolean formula in n variables and m clauses in CNF, find a satisfying assignment, if any, such that in every clause exactly one literal is true.

1. Give a reduction from EXACT SAT to MIN EXACT HITTING SET which is defined as follows. Given a collection $\mathcal{F}$ of subsets of a finite universe $\mathcal{U}$, find a subset $X \subseteq \mathcal{U}$ of minimum size such that every set in $\mathcal{F}$ contains exactly one element of X.

2. Show that the MIN EXACT HITTING SET problem can be solved in time $O^*(2^m)$, where $m = |\mathcal{F}|$.

This will show that EXACT SAT can be solved in time $O^*(2^m)$ time.

Homework Assignment H14 (10 Points) LONGEST COMMON SUBSEQUENCE OF 3 STRINGS. Given three strings $\mathbf{x}, \mathbf{y}, \mathbf{z}$ over a finite alphabet Σ, design an algorithm that finds out the longest common subsequence of these strings. What is the running time of your algorithm?