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Tutorial 2
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Tutorial Exact Algorithms

Exercise T3

Let F be a formula in CNF. A clause $\{l_1, \dots, l_k\}$ is exactly satisfied if exactly one of the literals is true. An assignment satisfies F exactly, if each clause is exactly satisfied.

Find an algorithm that decides whether F can be exact satisfied in time $O^*(\tau(3,3,3)^n)$.

Exercise T4

The problem Set Packing is defined as follows:

Input:	A family $\mathcal{S} = \{S_1, \dots, S_m\}$ of sets.
Question:	Is there a subset $S' \subset \mathcal{S}$ of size at least k such that $S_i \cap S_j = \emptyset$ for all $S_i, S_j \in S'$ with $i \neq j$?

Design an algorithm that solves this problem in $O^*(1.47^m)$.

Homework Assignment H3 (10 Points)

The problem Hitting Set is defined as follows:

Input:	A family $\mathcal{S} = \{S_1, \dots, S_m\}$ of subsets of a universe $\mathcal{U}$.
Question:	Is there a $U \subseteq \mathcal{U}$ of size at most k such that $U \cap S_i \neq \emptyset$ for all $S_i \in \mathcal{S}$

Design an algorithm that solves this problem in at most $O^*(\sqrt{2}^{|\mathcal{U}|+|\mathcal{S}|})$ steps. Hint: This problem is similar to Set Cover.

Homework Assignment H4 (10 Points)

Give a formal proof for the following lemmas:

Lemma 1 *Let $G = (V, E)$ be a graph and $v \in V$ with $\deg(v) = 1$. Then*

$$\alpha(G) = \alpha(G \setminus \{v\}) + 1.$$

Lemma 2 *Let $G = (V, E)$ and let G consist of connected components $G_1, \dots, G_l$. Then*

$$\alpha(G) = \sum_{i=1}^{l} \alpha(G_i).$$

Lemma 3 *Let $G = (V, E)$ be a graph and $\Delta(G) \leq 2$. Then $\alpha(G)$ can be computed in polynomial time.*