RWTH Aachen **WS 2025**
Theoretical Computer Science Exercise 2
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Exercise for Analysis of Algorithms

Exercise 5

If a flow diagram consists of n nodes and m edges, how many fundamental cycles do we get?

Exercise 6

Prove or disprove: In every flow diagram you can find a spanning tree such that all fundamental cycles contain only edges that are labeled with plus.

Exercise 7

In this exercise, we consider Prim's Algorithm, which computes a minimum spanning tree. The input to this algorithm is a graph $G = (V, E)$, a weight function on the edges $w : E \to \mathbf{R}$ and a starting node r.

for each $u \in V$ **do**
    $key[u] \leftarrow \infty$
    $\pi[u] \leftarrow$ NIL
$key[r] \leftarrow 0$
$M \leftarrow V$
while $(M \neq \emptyset)$ **do**
    $u \leftarrow$ min-from(M)
    for each $v \in$ neighbors(u) **do**
        if $(v \in M) \wedge (w(u, v) < key[v])$ **then**
            $\pi[v] \leftarrow u$
            $key[v] \leftarrow w(u, v)$

Construct the control flow graph, a spanning tree in the control flow graph, the fundamental cycles, a corresponding linear system of equations and a solution to this system.

Exercise 8

Let $w \in \{a, b\}^n$ a word that has been chosen uniformly at random. How often is the body of the `while`-loop executed on average in the following algorithm? The function `is_palindrome` tests whether a word is a palindrome, i.e., the same when read backwards.

```
i = 2;
while (i <= n)
  if (is_palindrome(w[1],...,w[i]))
     return true;
  i++;
return false;
```

Exercise 9

RWTH Aachen University has an exclusive contract with the well-known *Uranus Corporation* on the delivery of canal and pump supplies. Unfortunately, the responsible person failed to notice that Uranus sells only the "one-fits-all" product KKuRPSE (Kanalkopplungsundregulierungspumpstationeinheit). Such a KKuRPSE is a cylinder with four connectors placed north, east, south, and west of the cylinder (see the figure).

Now, the Institute for Tunnel and Canal Construction (ITCC) is conducting some research: The researchers first place n KKuRPSEs on a big green. Then they start to connect KKuRPSEs as follows: They digg a new canal between two arbitrary connectors. The new canal cannot cross another existing canal, of course. They then place a new KKuRPSE somewhere into this new canal, such that exactly two connectors on opposite sides of the new KKuRPSE are now attached to the canal.

The question is: How often can this connection procedure be repeated depending on the number n of initial KKuRPSEs on the green?

Example

Assume the researchers start with two KKuRPSEs, here labeled 0 and 1. Then a couple of connection steps are executed, where each new KKuRPSE receives consecutive numbers until no more connections are possible.