Analysis of Algorithms, WS 2020
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Exercise Sheet 08

The Gardener's Tale:

Just a christmas tradition
a gift exchange of sorts
a present for a person
and one rule to enforce

Each of us buys a present
only one and no more
who is your secret santa?
yourself? that'd be a bore!

We have to write our names
on a little piece of paper
one at random you may choose
what have we got to loose?

Out of n persons and with great care
how many would we expect to despair?

Having chosen themselves
as secret santa instead
their own name they read
and restart again with dread

Out of n persons and with great caution
Many times we give the names,
yes, but how often?

The Politician's Tale:

If you want to liven up your home
With lovely decoration
You may want to set a tone
To raise your loved ones expectations

Use christmas ornaments and balls
But beware of superstition
'Cause even the best decorator falls
If others detect repetition

If you are color blind
And still want to decorate
Be sure not to find
Any ornaments to replicate

Old christmases display
Not any of the n
For there is dismay
For more than a span

The Physicist's Tale:

There once was a man in postal office
Who always read a book while he drank his coffee
But one day the novel he liked went missing
So instead he read letters about lovers kissing
On the first letter Jane wrote to Austin
Who was in a lecture on the Higgs boson
The second letter opened led to a third
And be before he knew it n letters were read
The letters and the envelopes were all so scattered
The empty cup of coffee seemed to hardly matter
Now opening letters could send him to prison
He had to find a way to find rhyme and reason
He figured he could choose a letter at random
And prayed that the cover was the right one
So what are the chances, what are the odds
That no single letter went to the right spot?

The Bicycle Repairman's Tale:

Long days — Stifling air
Mediocre food — Drinking Mate
Full of ideas — Familiar faces
Have become friends — Brothers in arms
Against weekly problems — The world can go on
When we'll be finally together
At the computer science center

The Policeman's Tale:

Ashore n sailors slunk,
and got awfully drunk.
When they returned,
they were unconcerned,
fell asleep in a random bunk.

The Lockpick's Tale:

n criminals want to fix
their finances by using a mix
their crypto to blend
for them then to spend
and not to get caught with their tricks

The Thief's Tale:

It needs to be fixed
Where its place is fine?
This poem will be mixed
Is there a line

Problem T18

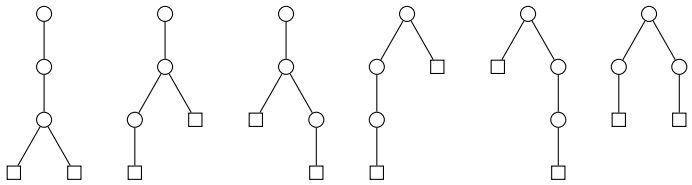
A permutation with no fixpoints is called a *derangement*. What is the probability that a random permutation of n items is a derangement? If a program generates random permutations, how long do you have to wait on average until you get a derangement? Before attacking this problem mathematically try your best judgement: Is the probability high or low?

Problem T19

Find a bivariate generating function and a closed-form expression for the number of bitstrings of length n that contain exactly m ones and do not contain the substring 11.

Problem H18 (10 credits)

Find a bivariate generating function for the number of (oriented) trees with exactly n internal and m external vertices $T_{n,m}$. For what values of n, m do we have $T_{n,m} = T_{m,n}$?
Example: $b_{3,2} = 6$ and these are the six trees with 3 internal and 2 external nodes:



As a warmup exercise try to find and draw all trees with 2 internal and 3 external nodes.

Hint: Do not do all the computations by hand. Seek the help of a computer algebra system. `maxima` or `WolframAlpha` can solve quadratic equations and can find the coefficients of a generating function via Taylor expansion.

Problem H19 (10 credits)

Use the symbolic method to calculate the number of words of length n that can be created by the following grammar. Emojis are the terminal symbols and capital letters are variables.

$$P \to \quad 😀 P 😍 \quad | \quad 🤡 P 🐺 \quad | \quad 💩 \quad | \quad 💩 P$$

Problem H20 (10 credits)

Use the symbolic method to calculate the number of words of length n that can be created by the following grammar with the starting Symbol P.

$$Q \quad \to \quad 😀 P 😍$$

$$P \quad \to \quad PQ \mid \varepsilon$$

Hint: Use the sequence operator.