Analysis of Algorithms, WS 2020
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Date: 25 November 2020

Exercise Sheet 05

Problem T11

Let $x \in \mathbf{R}^+$. Is $\lceil \sqrt{x} \rceil = \left\lceil \sqrt{\lceil x \rceil} \right\rceil$?

Problem T12

Use summation factors to solve the following recurrence:

$$\begin{aligned} a_0 &= 0 \\ a_n &= \frac{a_{n-1}}{n} + \frac{1}{(n-1)!} \qquad \text{for } n \geq 1 \end{aligned}$$

Problem T13

Consider the following program:

```
int sel_sort ( int a[], int n ) {
  for ( int i = 0; i < n; ++i ) {
    int min = i;
    for ( int j = i; j < n; ++j ) {
      if ( a[j] < a[min] ) {
        min = j;
      }
    }
    int temp = a[i];
    a[i] = a[min];
    a[min] = temp;
  }
}
```

The input to this program is an array $a[0, \ldots, n-1]$ that contains n pairwise distinct integer keys in random order.

a) Explain how this program sorts the given array.

b) Analyse how often each instruction of the program is executed on average depending on n.

c) There is only one instruction whose analysis is not trivial. Which one is it?

   Make a table for small values of n by hand that lists the results for this instruction. Compare the table entries with the results from your closed formula that you obtained in b).

Problem H10 (10 credits)

Solve the following recurrence relation by order reduction:

$$a_0 = 0 \qquad a_1 = 1 \qquad a_{n+2} + a_{n+1} - n^2 a_n = n!$$

This is the same recurrence relation as in T10, but the initial conditions are different. That exercise should be solved by order reduction. Solve this exercise by whatever method you like.

Problem H11 (15 credits)

Consider the following algorithm that searches an element x in a sorted array a of length $n = km + 1$:

```
i:= 1;
while a[i]<=x
  if a[i]=x then return i;
  i:=i+m;
  if i>n return 0;
for j=i-1 downto max(1,i-(m-1))
  if a[j]=x then return j;
if a[j]<x then return 0;
return 0;
```

a) Draw the search tree and compute the internal and external path length for $n = 10$ and $m = 3$.

b) Determine C^+ and C^- for arbitrary m, k.

c) What is, for given n, the best choice for m w.r.t. the running time?

Problem H12 (15 credits)

Generalize the question in T11 to functions that are monotonically increasing, continuous, and do not map non-integers to integers.