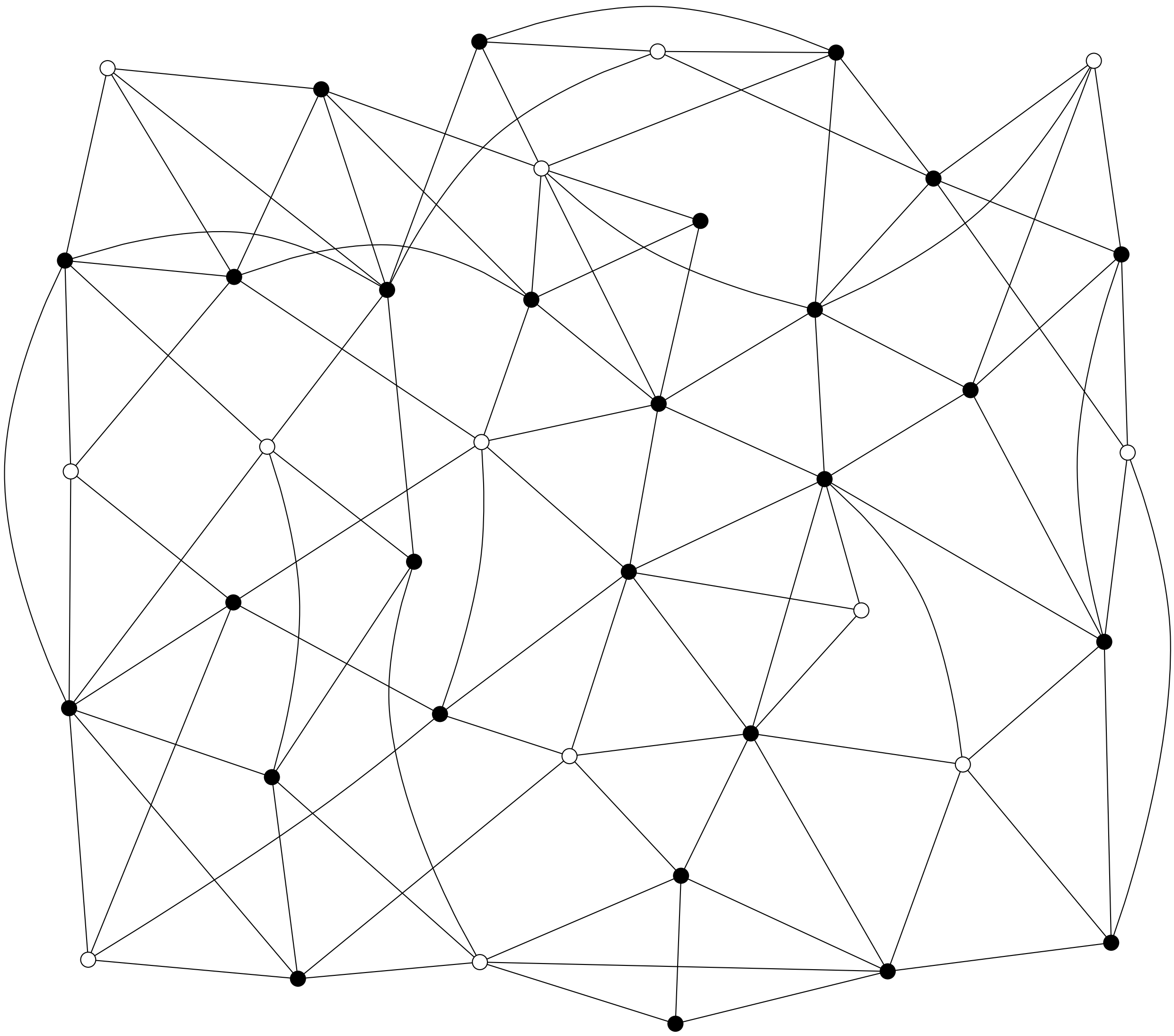
Analysis of Algorithms, WS 2020
Prof. Dr. P. Rossmanith
Dr. E. Burjons, H. Lotze, D. Mock

Date: 11 November 2020

Exercise Sheet 03

Problem T7

A *vertex cover* of an undirected graph $G = (V, E)$ is a subset $C \subseteq V$ of its vertices such that at least one endpoint of every edge is in C, i.e., for every $(v_i, v_j) \in E$, either $v_i \in C$ or $v_j \in C$. Informally speaking, "C covers all edges." It is an NP-complete problem to find out whether a graph has a vertex cover of a given size. The following example shows a graph of order 40. The black vertices comprise a minimum size vertex cover.



Now, given a graph $G = (V, E)$ we define its size as the number of vertices $|V|$ of the graph. So, let us consider a graph of size n and let k be targeted vertex cover size. Find an algorithm for Vertex Cover that runs in time $1.5^k n^{O(1)}$. Hints: If a graph has maximum degree two, i.e., without any vertex of degree three or more, this problem can be solved in polynomial time. On the other hand, if a vertex v is not in the vertex cover, all of its neighbors have to be there.

Problem T8

Solve the recurrence relation

$$a_n = 5a_{n-1} - 8a_{n-2} + 4a_{n-3}$$

with the starting conditions $a_0 = 1$, $a_1 = 3$, $a_2 = 8$.

Problem H6 (10 credits)

Solve the following recurrence: Let $a_0 = 1$, $a_1 = 1$, $a_2 = 4$ and

$$a_n = 2a_{n-1} - a_{n-2} + 2a_{n-3}, \text{ for } n \geq 3.$$

Problem H7 (10 credits)

Our task is to generate a word of length n over the alphabet $\{0, 1\}$ which contains neither two consecutive zeros nor three consecutive ones.

Daniel proposes the following algorithm: The algorithm generates a word of length n uniformly at random. If the word fulfills the property, it is returned. Otherwise, the algorithm tries again until it finds one.

What is the expected number of rounds the algorithm needs? Consider this in particular for 32bit words.

0110110110110101	0101101101011010	0101010101011011	1101010110101010	1010110110110110
0110110110110110	0101101101011011	0101010101010101	1101010110101011	1010110110101101
0110110110101101	0101101101010101	0101010101010110	1101010101101101	1010110110101010
0110110110101010	0101101101010110	1101101101101101	1101010101101010	1010110110101011
0110110110101011	0101101011011010	1101101101101010	1101010101101011	1010110101101101
0110110101101101	0101101011011011	1101101101101011	1101010101011010	1010110101101010
0110110101101010	0101101011010101	1101101101011010	1101010101011011	1010110101101011
0110110101101011	0101101011010110	1101101101011011	1101010101010101	1010110101011010
0110110101011010	0101101010110101	1101101101010101	1101010101010110	1010110101011011
0110110101011011	0101101010110110	1101101101010110	1011011011011010	1010110101010101
0110110101010101	0101101010101101	1101101011011010	1011011011011011	1010110101010110
0110110101010110	0101101010101010	1101101011011011	1011011011010101	1010101101101101
0110101101101101	0101101010101011	1101101011010101	1011011011010110	1010101101101010
0110101101101010	0101011011011010	1101101011010110	1011011010110101	1010101101101011
0110101101101011	0101011011011011	1101101010110101	1011011010110110	1010101101011010
0110101101011010	0101011011010101	1101101010110110	1011011010101101	1010101101011011
0110101101011011	0101011011010110	1101101010101101	1011011010101010	1010101101010101
0110101101010101	0101011010110101	1101101010101010	1011011010101011	1010101101010110
0110101101010110	0101011010110110	1101101010101011	1011010110110101	1010101011011010
0110101011011010	0101011010101101	1101011011011010	1011010110110110	1010101011011011
0110101011011011	0101011010101010	1101011011011011	1011010110101101	1010101011010101
0110101011010101	0101011010101011	1101011011010101	1011010110101010	1010101011010110
0110101011010110	0101010110110101	1101011011010110	1011010110101011	1010101010110101
0110101010110101	0101010110110110	1101011010110101	1011010101101101	1010101010110110
0110101010110110	0101010110101101	1101011010110110	1011010101101010	1010101010101101
0110101010101101	0101010110101010	1101011010101101	1011010101101011	1010101010101010
0110101010101010	0101010110101011	1101011010101010	1011010101011010	1010101010101011
0110101010101011	0101010101101101	1101011010101011	1011010101011011	
0101101101101101	0101010101101010	1101010110110101	1011010101010101	
0101101101101010	0101010101101011	1101010110110110	1011010101010110	
0101101101101011	0101010101011010	1101010110101101	1010110110110101	