RWTH Aachen
Theoretical Computer Science
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WS 2017
Exercise 10
16.1.2018

Exercise for Analysis of Algorithms

Today's tutorial will be based on the following GF:

$$U(z) := \frac{1 - z - \sqrt{(1-3z)(1+z)}}{2z}.$$

Useful values of the Gamma function:

x	$-\frac{5}{2}$	$-\frac{3}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{5}{2}$
$\Gamma(x)$	$-\frac{8\sqrt{\pi}}{15}$	$\frac{4\sqrt{\pi}}{3}$	$-2\sqrt{\pi}$	$\sqrt{\pi}$	$\frac{\sqrt{\pi}}{2}$	$\frac{3\sqrt{\pi}}{4}$

Exercise T22

Prove that $[z^n]U(z) = 3^n n^{O(1)}$.

Exercise T23

Let $V(z) := -\sqrt{(1-3z)(1+z)}$. What is the relation of $[z^n]V(z)$ and $[z^n]U(z)$?

Exercise T24

Compute $W(z) := V(z) + 2/\sqrt{3}\,\sqrt{1-3z}$ and estimate $[z^n]W(z)$ *as good as possible.*

Exercise T25

What follows for $[z^n]V(z)$?

Exercise H15

Prove that

$$\binom{-r}{n} = (-1)^n \binom{r+n-1}{n}$$

for $r \in \mathbf{R}, n \in \mathbf{Z}$.

Exercise H16

Prove that

$$[z^n](1-z)^w \sim \frac{n^{-w-1}}{\Gamma(-w)}$$

for $w \in \mathbf{C}$ without using the theorem of the lecture. (The idea of this assignment is to get a deeper insight into the theorem.)
Hint: Use Newton's formula, then the first exercise on this sheet. Now replace the binomial coefficient by factorials or the gamma function. In the first case, you need to be careful with a definition of factorials for real numbers. In general, however, $\Gamma(n+1) = n!$.

Exercise H17

Determine $[z^n]U(z)$ in the form $h_n + o(3^n n^{-7/2})$, where h_n shall be in closed form.