

### Exercise for Analysis of Algorithms

#### Exercise T7

Given an array  $a$  of length  $n$ , an algorithm compares all pairs  $(a[i], a[j])$  for all  $i < j \leq n$ , and then calls itself recursively on all proper prefixes of  $a$ .

How often does the algorithm compare two pairs? Use the repertoire method!

#### Exercise T8

Solve the following recurrence: Let  $a_0 = 1$ ,  $a_1 = 1$ ,  $a_2 = 4$  and

$$a_n = 2a_{n-1} - a_{n-2} + 2a_{n-3}, \text{ for } n \geq 3.$$

#### Exercise H4

Solve the following recurrence: Let  $a_0 = 0$ ,  $a_1 = 3$  and

$$a_n = 4a_{n-1} - 4a_{n-2} \text{ for } n > 1.$$

#### Exercise H5

Use the repertoire method to find a closed form for the following recurrence:

$$\begin{aligned} a_0 &= 5 \\ a_1 &= 9 \\ a_n &= na_{n-1} + n^2a_{n-2} - n^4 - 3n^2 + 5 \quad \text{for } n \geq 2 \end{aligned}$$

#### Exercise H6

Solve the following recurrence and find a nice representation of the solution (in a mathematical sense).

$$\begin{aligned} c_0 &= 2 \\ c_1 &= 4 \\ c_n &= c_{n-2}^{\log c_{n-1}} \end{aligned}$$

Hint: Let  $F_n$  be the  $n$ th Fibonacci number. Write  $c_n$  as some function of  $F_n$ .