

Analysis of Algorithms — Tutorial

Exercise 11-1

Sort the following generating functions *within one minute* by their exponential growth!

1. $A(z) = 1/\sqrt{1-z/2}$

2. $B(z) = (1+z)/(1-z)$

3. $C(z) = \frac{1}{1-e^{-z-1/3}}$

Exercise 11-2

An algorithm is given an array of length $n \geq 0$ and, if $n \geq 2$, for each $1 \leq k \leq n$ calls itself on some random subarray of length k with probability $\frac{1}{2}$. Compute the exponential growth of the running time of this algorithm.

Exercise 11-3

Determine $[z^n] \frac{1}{2-e^z}$ up to an additive error of $O(12^{-n})$!

Homework Assignment 11-1 (10 points)

Determine the exponential growth of $[z^n]G(z)$, where

a) $G(z) = z^2/(1-z-z^2)$,

b) $G(z) = \sqrt{1+2z} - \sqrt{2+2z-4z^2}/\sqrt{3}$,

c) $G(z) = \ln(1+\sin(z))/\ln(1+\cos(z))$.

Homework Assignment 11-2 (10 points)

$$A(z) = \frac{\sqrt{1-z^7}}{2z^2-3z+1} \quad B(z) = \frac{1-z^2}{e^{z+3z^2}} \quad C(z) = z^5 + 3z^2(z^3 + z^2 + 8)$$

Sort the sequences a_n , b_n , and c_n by their exponential growth.

Homework Assignment 11-3 (10 Points)

Determine $g_n = [z^n]G(z)$ up to an additive error of $O(4^n)$, where

$$G(z) = \sum_{n=0}^{\infty} g_n z^n = \frac{15z^2 + 8z + 1}{15z^2 - 8z + 1}.$$